

Productivity and Quality of Multi-product Firms

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Motivation

- ▶ In the literature of firm-level analysis using production data, it is usually (implicitly) assumed that each firm produces a single product and focuses on **across-firm heterogeneity**.
- ▶ In reality, many firms produce more than one product, potentially associated with different levels of quality and productivity – **within-firm heterogeneity**.
- ▶ Many questions can only be answered after firm-product level productivity and quality are estimated:
 - ▶ Does a firm's **core competence** lie in productivity or quality (or both)?
 - ▶ Is there a complementarity or substitution (**trade-off**) between productivity and quality within firms?
 - ▶ How does **intra-firm resource reallocation** shape the distributions of productivity and quality at the firm level?

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Challenges

Estimating productivity and output quality at the firm-product level is challenging:

- ▶ **Data/measurement:** need firm-product level output, input, and prices, but
 - ▶ cannot observe input allocation in the production of different products;
 - ▶ rarely observe intermediate input prices (even at firm-level).
- ▶ **Methodological:** high-dimensional unobservable heterogeneity makes it challenging to directly use proxy-based approach (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Akerberg et al., 2015; and Gandhi et al., 2016).

Recent development for multi-product firm estimation methods:

- ▶ **Transformation function** with proxy-based approach (Dhyne, Petrin, Smeets, and Warzynski, 2022) estimates product productivity after controlling for “other products”;
- ▶ **Product production function** with imputed input shares from **firm optimization conditions** (Orr, 2022; Valmari, 2022; Chen and Liao, 2020).

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Our Approach

1. Use a dataset of manufacturing industries in Mexico containing:
 - ▶ firm-level inputs (labor, materials, and capital);
 - ▶ firm-product level output prices and quantities (more than 15 products).
2. Develop an empirical model to uncover productivity and quality (instead of imputing input shares), built on Grieco, Li, and Zhang (2016, 2022), and Li and Zhang (2022):
 - ▶ a transformation function with CES production component;
 - ▶ profit-maximizing firms (to use optimization conditions).
3. Application:
 - ▶ trade-off between productivity and quality, i.e., cost of quality;
 - ▶ counterfactual exercise on contribution of decrease in cost of quality in terms of overall productivity and within-firm resource reallocation.

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Advantages of Our Approach

- ▶ The empirical model can address the traditional challenges of:
 - ▶ requirement of instrumental/proxy variables (for **high dimension** of productivity and quality);
 - ▶ unobserved firm-level heterogenous **intermediate input prices**;
 - ▶ unobserved **input allocation and potential input sharing** across products.
- ▶ We allow for (but do not impose in estimation):
 - ▶ trade-off between productivity and quality;
 - ▶ flexible interdependence in dynamic evolution of productivity.

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Demand

In each period t , there are J firms (indexed by j) in the industry.

There are N (horizontal) categories of products (e.g., kids shoes and women's shoes) that a firm can choose to produce: $n = 1, \dots, N$.

For category n , firm j produces **one variant** with its choice of quality Ξ_{jtn} .

The set of products of firm j is Λ_{jt} (e.g., women's shoes and men's shoes).

For each category n , a representative consumer values both quality and quantity of the products (or, equivalently, **quality-adjusted product**) offered by the firms:

$$U_{tn} = \left[\sum_j \left(\Xi_{jtn}^{\frac{1}{\eta_n - 1}} Q_{jtn} \right)^{\frac{\eta_n - 1}{\eta_n}} \right]^{\frac{\eta_n}{\eta_n - 1}} .$$

Denote $\xi_{jtn} = \ln \Xi_{jtn}$.

Demand

Consumer's utility maximization problem implies the demand function (in log):

$$\ln Q_{jtn} = -\eta_n \ln P_{jtn} + \tilde{\xi}_{jtn},$$

where η_n is the elasticity of demand and $\tilde{\xi}_{jtn} = \xi_{jtn} + \phi_{tn} + \psi_{jn} + v_{jt}$.

- ▶ ϕ_{tn} : a product-specific expenditure shifter that depends on macroeconomic conditions (consumer income and market sizes);
- ▶ ψ_{jn} : a firm-product factor capturing consumers' subjective tastes, brand images, and product measurement units;
- ▶ v_{jt} : firm-year demand heterogeneity such as marketing.

Denote $\chi_{jtn} = \phi_{tn} + \psi_{jn} + v_{jt}$.

Production – Functional Form

Given the set of products (Λ_{jt}) and associated quality (ξ_{jtn} , $n \in \Lambda_{jt}$), the firm uses labor (L_{jt}), material (M_{jt}), and capital (K_{jt}) to produce output quantity (Q_{jtn} , $n \in \Lambda_{jt}$) following a CES transformation function:

$$\sum_{n \in \Lambda_{jt}} e^{-\tilde{\omega}_{jtn}} Q_{jtn} = F(L_{jt}, M_{jt}, K_{jt}) \equiv \left[\alpha_L L_{jt}^\gamma + \alpha_M M_{jt}^\gamma + \alpha_K K_{jt}^\gamma \right]^{\frac{\rho}{\gamma}}.$$

- ▶ L_{jt} and M_{jt} are flexibly chosen by the firm.
- ▶ Inputs are costless transferable across the production of different products, but no assumption on input allocation (or sharing) across products.
- ▶ Parameter ρ governs returns to scale.
- ▶ Rate of substitution across products determined by relative value of $\tilde{\omega}_{jtn}$.

Production – Productivity and Cost of Quality

Quantity-based productivity at the firm-product level: $\tilde{\omega}_{jtn}$

- ▶ Trade-off between quality and quantity (Grieco and McDevitt, 2017):

$$\tilde{\omega}_{jtn} = \omega_{jtn} - h(\xi_{jtn}),$$

where ω_{jtn} is *technical efficiency* and $h(\xi_{jtn})$ is *cost of quality*.

- ▶ Producing high-quality products is more costly (requiring more production procedures/specialized machinery/higher quality materials)
→ lower quantity output (thus productivity), holding inputs fixed.

Static Decisions: Inputs and Outputs

At the beginning of period t , the firm observes the state

$$s_{jt} = (\Lambda_{jt}, \omega_{jt}, \xi_{jt}, K_{jt}, P_{Mjt}, P_{Ljt}, \chi_{jt}).$$

The firm's static problem is to maximize its period profit, by optimally choosing M_{jt} , L_{jt} , and $\mathbf{Q}_{jt} = \{Q_{jtn}\}$, $n \in \Lambda_{jt}$.

Specifically, the period (static) profit is written as:

$$\pi(s_{jt}) = \max_{\mathbf{Q}_{jt}, M_{jt}, L_{jt}} \sum_{n \in \Lambda_{jt}} P_{jtn} Q_{jtn} - P_{Mjt} M_{jt} - P_{Ljt} L_{jt}$$

subject to: demand and production functions,

where P_{Mjt} and P_{Ljt} are the material price and wage rate, respectively. Importantly, they can be different across firms and varying over time.

Dynamic Decisions – Products, Quality, and Productivity

In the end of each period t , the firm chooses the product set, quality levels, and efficiency-improvement investment for the next period $t + 1$.

These decisions are made conditional on the current state and after observing adjustment costs related to product set and quality levels.

(The dynamic decisions are simply for the conceptual completion of the model – we do not use these dynamic decisions nor estimating the dynamic model.)

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Estimation of the Empirical Static Model – Challenges

- ▶ Productivity and quality **endogenously** influence input and output choices;
- ▶ **High dimension** of productivity and quality at firm-product level would require a large number of IVs;
- ▶ No information on **allocation nor sharing of inputs** (across products);
- ▶ Researchers observe material expenditure ($E_{Mjt} = P_{Mjt}M_{jt}$) but not material quantity and prices separately, while **material prices are heterogenous** – ignoring this will cause estimation bias (Ornaghi, 2006, Grieco, Li, Zhang, 2016).

Estimation of the Empirical Static Model – Solution

Idea: Because firm's choices are made according to the profit maximisation problem, we can **invert the first order conditions to recover unobservable variables** as functions of parameters, firm's choices, and observable information.

Step 1: use

- ▶ observable data: $(L_{jt}, E_{Ljt}, E_{Mjt}, K_{jt}, \mathbf{Q}_{jt}, \mathbf{P}_{jt})$, and
- ▶ invertible first-order conditions of profit maximization,

to establish a **one-to-one mapping** to:

- ▶ unobservable variables: $(\tilde{\xi}_{jt}, \tilde{\omega}_{jt}, M_{jt}, P_{Mjt}, \lambda_{jt})$, where λ_{jt} is the Lagrangian.

Step 2: **substitute** the recovered unobservables into the production function to form an estimating equation to estimate underlying parameters.

Step 3: **compute directly** $\tilde{\xi}_{jt}$ and $\tilde{\omega}_{jt}$ from the estimated mapping.

Example

An accounting exercise of a two-product case:

- ▶ 7 unobservable variables: $(\tilde{\xi}_{jt1}, \tilde{\xi}_{jt2}, \tilde{\omega}_{jt1}, \tilde{\omega}_{jt2}, M_{jt}, P_{Mjt}, \lambda_{jt})$;
- ▶ 7 unique equations, including:
 - ▶ 2 FOCs for labor and material;
 - ▶ 2 FOCs for the quantities of the two products;
 - ▶ 2 demand functions;
 - ▶ 1 identity equation: $E_{Mjt} = P_{Mjt}M_{jt}$.

These equations **always** admit a unique mapping from observables to unobservables.

Then, we can substitute the unobservables in the production function to estimate parameters.

Note: the number of FOCs of product quantity **increases with the number of products** → solve the availability problem in the traditional proxy approach.

First-order Conditions

- ▶ The first-order conditions with respect to labor and materials are:

$$P_{Ljt} = \lambda_{jt} \frac{\partial F}{\partial L_{jt}}, \quad P_{Mjt} = \lambda_{jt} \frac{\partial F}{\partial M_{jt}}.$$

- ▶ The first-order condition with respect to Q_{jtn} is:

$$P_{jtn} = \underbrace{\frac{\eta_n}{\eta_n - 1}}_{\text{markup}} \underbrace{\lambda_{jt} e^{-\tilde{\omega}_{jtn}}}_{\text{marginal cost}}, \quad \forall n,$$

where λ_{jt} is the Lagrangian multiplier.

Intuition: price is markup-adjusted marginal cost which consists of

- ▶ a firm-level component (λ_{jt});
- ▶ a firm-product-level component ($\tilde{\omega}_{jtn}$).

→ **across-firm variation** in observable cost data identifies firm-level λ_{jt} ; conditional on the firm (λ_{jt}), **the price variation within the firm identifies the intra-firm productivity ($\tilde{\omega}_{jtn}$) differences.**

Solving for the Unobservables

These first-order conditions **always admit a unique solution** for the unobservable variables:

$$\blacktriangleright P_{Mjt} = \left[\frac{\alpha_M}{\alpha_L} \right]^{\frac{1}{\gamma}} \left[\frac{E_{Mjt}}{E_{Ljt}} \right]^{1 - \frac{1}{\gamma}} P_{Ljt};$$

$$\blacktriangleright \tilde{\xi}_{jtn} = \ln Q_{jtn} + \eta_n \ln P_{jtn};$$

$$\blacktriangleright \tilde{\omega}_{jtn} = \ln \left\{ \frac{\eta_n}{(\eta_n - 1)P_{jtn}} \frac{E_{Ljt}}{\rho \alpha_L L_{jt}^\gamma} \underbrace{\left[\alpha_L L_{jt}^\gamma \left(1 + \frac{E_{Mjt}}{E_{Ljt}} \right) + \alpha_K K_{jt}^\gamma \right]^{1 - \frac{\rho}{\gamma}}}_{\lambda_{jt}} \right\}.$$

All variables on the right-hand side are observables, and no separate input allocation is needed.

Main Estimating Equation

Production function with an i.i.d. transitory (**unexpected**) error u_{jt} :

$$\sum_{n \in \Lambda_{jt}} e^{-\tilde{\omega}_{jtn}} Q_{jtn} = \left[\alpha_L L_{jt}^\gamma + \alpha_M M_{jt}^\gamma + \alpha_K K_{jt}^\gamma \right]^\frac{\rho}{\gamma} e^{u_{jt}}.$$

Substitute the unobservables into the above to obtain estimating equation:

$$\ln \left[\sum_{n \in \Lambda_{jt}} \frac{(\eta_n - 1)\rho}{\eta_n} R_{jtn} \right] = \ln \left[E_{M_{jt}} + E_{L_{jt}} \left(1 + \frac{\alpha_K}{\alpha_L} \left(\frac{K_{jt}}{L_{jt}} \right)^\gamma \right) \right] + u_{jt},$$

where R_{jtn} is product revenue.

- ▶ Advantage: only unexpected shock u_{jt} is unobservable.
- ▶ Remaining issues:
 - ▶ ρ and η_n are not separately identified;
 - ▶ u_{jt} is in revenue and thus is correlated with R_{jtn} – need IVs (via GMM);

Addressing the Remaining Issues

Intuitively, the reason of the non-identification (between ρ and η_n) is that only revenue data (instead of quantity and price) are used in the estimation.

Traditional solution: directly estimate the demand function:

$\ln Q_{jtn} = -\eta_n \ln P_{jtn} + \tilde{\xi}_{jtn}$, treating $\tilde{\xi}_{jtn}$ as an error term.

Advantage: with $\hat{\eta}_n$, our equation can be conveniently estimated via NLLS:

$$\ln \left[\sum_{n \in \Lambda_{jt}} \frac{(\hat{\eta}_n - 1)}{\hat{\eta}_n} R_{jtn} \right] = -\ln \rho + \ln \left[E_{M_{jt}} + E_{L_{jt}} \left(1 + \frac{\alpha_K}{\alpha_L} \left(\frac{K_{jt}}{L_{jt}} \right)^\gamma \right) \right] + u_{jt}.$$

Problem: hard to find (firm-product-level or even firm-level) IVs for P_{jtn} to estimate demand function, because:

- ▶ commonly used IVs (cost shifter) is correlated with the output quality level if there is a cost of producing high quality.

Our Solution – Take Advantage of Multiple Products

We use the firm's optimal decision on balancing the sales of different products in profit maximisation.

That is, the **intra-firm** variation of sales of one product relative to another (conditional on everything else) provides information on how the elasticities of the two products differ (see also Grieco, Li and Zhang, 2022).

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Formally, denote product 1 as the reference product. Thus, the sales ratio of product 1 against product n implied by the FOCs gives:

$$\ln(R_{jt1}) = c_n + \frac{\eta_1 - 1}{\eta_n - 1} \ln(R_{jtn}) + \mu_{jtn}, \quad n = 2, \dots, N,$$

where

$$\mu_{jtn} = (\eta_1 - 1) \left[\underbrace{\left(\tilde{\omega}_{jt1} + \frac{1}{\eta_1 - 1} \tilde{\xi}_{jt1} \right) - \left(\tilde{\omega}_{jtn} + \frac{1}{\eta_n - 1} \tilde{\xi}_{jtn} \right)}_{\text{difference in quality-adjusted productivity}} + \underbrace{\frac{\eta_1 - \eta_n}{(\eta_1 - 1)(\eta_n - 1)} u_{jt}}_{\text{measurement error component}} \right]$$

and c_n is a product constant.

We are interested in $\frac{\eta_1 - 1}{\eta_n - 1}$, and firm-level IVs (e.g., K_{jt} , P_{Ljt} , and $\frac{E_{Mjt}}{L_{jt}}$) are sufficient:

- ▶ correlated with R_{jtn} (i.e., **absolute** revenue level of a product);
- ▶ but uncorrelated with μ_{jtn} (i.e., **relative** capability of producing a product over another).

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A Summary of Estimation Procedure

- ▶ Use 2SLS to estimate $\frac{\eta_1-1}{\eta_n-1}$ via a system of equations of sales ratios:
 - ▶ IVs: firm-level capital, wage rate, and material expenditure per worker;
 - ▶ this identifies (a function of) demand elasticities using the optimal allocation of products.
- ▶ Use GMM to estimate all other parameters (using estimated $\frac{\eta_1-1}{\eta_n-1}$ as constraints) via the main estimating equation:
 - ▶ IVs: material expenditure, labor, capital per worker, and material expenditure per worker;
 - ▶ this identifies the parameters of the input-output transformation function.
- ▶ Compute $\tilde{\omega}_{jt}$ and $\tilde{\xi}_{jt}$ after all parameters estimated.

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Data

Three large manufacturing industries in Mexico: footwear, paper and printing, and pharmaceuticals, during 1994-2007.

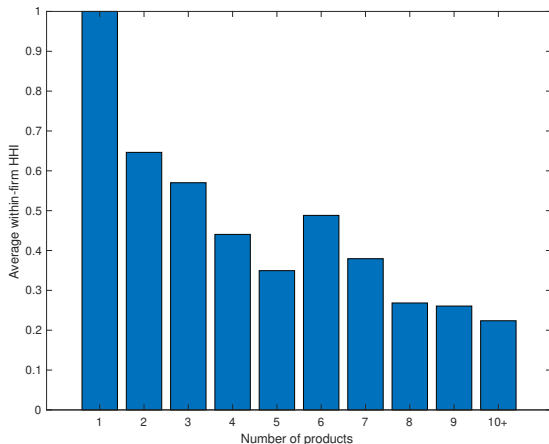
Variables: product prices and quantity at the firm-product level; inputs (material expenditure, labor expenditure, wage rate, capital stock) at the firm level.

Multi-product production is an essential feature of the data.

| | Footwear | Printing | Pharmaceutical |
|---------------------------------------|----------|----------|----------------|
| Firm-year observations | 617 | 692 | 858 |
| Total number of products | 4 | 14 | 16 |
| Mean number of products per firm | 1.4 | 4.3 | 7.0 |
| Mean number of firms per product-year | 21 | 19 | 43 |
| Share of MPFs | 0.208 | 0.554 | 0.846 |
| MPF revenue share | 0.389 | 0.599 | 0.940 |

Data: Within-firm HHI

All products are generally important for firms' total revenues — **genuine multi-product firms**.



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Demand Elasticity Estimates

| Parameter | Footwear | Printing | Pharmaceutical |
|-------------|-------------------|------------------|------------------|
| η_1 | 8.722 (1.956) | 4.306 (1.833) | 4.671 (1.778) |
| η_2 | 7.163 (2.279) | 8.189 (2.412) | 3.783 (1.758) |
| η_3 | 11.964 (3.538) | 4.220 (1.190) | 5.382 (2.518) |
| η_4 | 10.530 (3.480) | 6.931 (2.264) | 5.096 (2.795) |
| η_5 | | 4.235 (1.306) | 5.111 (3.176) |
| η_6 | | 4.537 (1.830) | 3.338 (1.157) |
| η_7 | | 4.884 (1.466) | 4.474 (2.089) |
| η_8 | | 5.839 (2.210) | 4.019 (1.606) |
| η_9 | | 6.760 (2.159) | 3.913 (2.392) |
| η_{10} | | 4.601 (1.409) | 4.090 (1.949) |
| η_{11} | | 6.332 (1.543) | 4.302 (2.235) |
| η_{12} | | 5.305 (1.550) | 3.796 (1.621) |
| η_{13} | | 4.077 (2.239) | 6.071 (2.681) |
| η_{14} | | 5.109 (1.348) | 9.575 (3.575) |
| η_{15} | | | 2.955 (2.445) |
| η_{16} | | | 3.471 (2.052) |

Production Function Parameter Estimates

| Parameter | Footwear | Printing | Pharmaceutical |
|------------|------------------|------------------|------------------|
| α_L | 0.195 (0.013) | 0.228 (0.016) | 0.228 (0.025) |
| α_M | 0.747 (0.047) | 0.671 (0.028) | 0.597 (0.068) |
| α_K | 0.058 (0.057) | 0.101 (0.037) | 0.175 (0.089) |
| σ | 1.069 (0.467) | 1.245 (0.131) | 1.185 (0.246) |
| ρ | 0.919 (0.119) | 1.097 (0.116) | 0.925 (0.118) |

What Determines Within-Firm Heterogeneity?

Table: Product rank (sales level), productivity and quality

| Dep. var.: | (1) | (2) | (3) | (4) |
|-------------------------|----------------------|----------------------|----------------------|----------------------|
| Log product rank, sales | All | Footwear | Printing | Pharmaceutical |
| Productivity | -0.724*** (0.133) | -1.996*** (0.466) | -0.761*** (0.175) | -0.729*** (0.235) |
| Quality | -0.190*** (0.048) | -0.207*** (0.056) | -0.183*** (0.059) | -0.220*** (0.066) |
| Firm-Product FE | yes | yes | yes | yes |
| Firm-Year FE | yes | yes | yes | yes |
| Product-Year FE | yes | yes | yes | yes |
| Observations | 9638 | 398 | 2981 | 6259 |
| R-squared | 0.893 | 0.947 | 0.918 | 0.877 |

Observations regarding **intra-firm** sales heterogeneity:

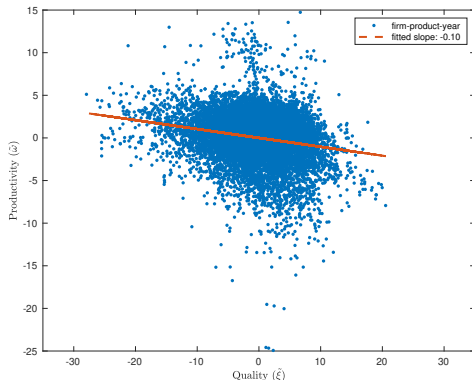
- ▶ Products closer to firms' core competence (i.e., with a lower rank value) have higher productivity and quality; nonetheless, productivity has a stronger impact than quality.
- ▶ Similar result on the basis of **sales growth**.

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Productivity and Quality Relationship

Products with high quality are associated with lower (quantity-based) productivity.



Cost of Quality

What is the trade-off between quality and productivity?

| Dep. var.: Productivity | (1) OLS | (2) OLS | (3) OLS | (4) IV1 | (5) IV2 | (6) IV3 |
|-------------------------|---------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Quality | -0.104** (0.047) | -0.181*** (0.033) | -0.200*** (0.038) | -0.186*** (0.035) | -0.203*** (0.047) | -0.186*** (0.035) |
| Firm FE | no | yes | no | no | no | no |
| Product FE | no | yes | no | no | no | no |
| Year FE | no | yes | no | no | no | no |
| Firm-Product FE | no | no | yes | yes | yes | yes |
| Firm-Year FE | no | no | yes | yes | yes | yes |
| Product-Year FE | no | no | yes | yes | yes | yes |
| Observations | 11021 | 11020 | 9638 | 8160 | 8160 | 8160 |
| R-squared | 0.026 | 0.721 | 0.998 | 0.821 | 0.821 | 0.821 |
| Kleibergen-Paap F | | | | 171.221 | 21.493 | 85.809 |
| Hansen J | | | | | | 0.123 |

Observations:

- ▶ Aligns with Grieco and McDevitt (2017) in healthcare industry; Li, Li, and Zhang (2023) in steel industry; Forlani et al (2023) in Belgian industries.
- ▶ Cost of quality higher for **more differentiated** and **newer** products.

Quality-adjusted Productivity and Quality

Taking both cost and benefit of quality into account: **quality-adjusted productivity**:

$$ATFP_{jtn} = \tilde{\omega}_{jtn} + \frac{1}{\eta_n - 1} \tilde{\xi}_{jtn}$$

— a comparable term of firm capability (Melitz, 2000).

Table: Within-firm relationship between ATFP and quality

| Quality | (1) All | (2) Footwear | (3) Printing | (4) Pharmaceutical |
|---------|---------------------|---------------------|---------------------|-----------------------|
| ATFP | 0.432*** (0.089) | 0.539*** (0.194) | 0.352*** (0.105) | 0.500*** (0.122) |

Observations:

- ▶ Positive relationship: firm-products with higher quality-adjusted productivity are associated with higher quality
 - consistent with endogenous quality choice models (Verhoogen, 2008; Kugler and Verhoogen, 2009, 2012; Hottman et al., 2016).

Outline

1. Model
2. Estimation Method
3. Data
4. Results
 - 4.1 What determines within-firm heterogeneity?
 - 4.2 What is the trade-off between productivity and quality?
 - 4.3 Counterfactual exercise: how costly is quality?
5. Conclusion

How Costly is Quality?

Positive **intra-firm** relationship between ATFP and quality implies:

- ▶ products with high ATFP → high quality (thus high cost) → high price → prevent resources from being allocated to high ATFP products;
- ▶ thus, reduction in cost of quality can increase productivity directly and indirectly (via intra-firm resource reallocation).

Counterfactual analysis:

- ▶ Reduce the cost responsiveness of quality by 1%. That is,
$$\text{counterfactual productivity} = \text{productivity} + 1\% \times \text{cost of quality}.$$
- ▶ Re-compute the optimal (static) decisions on inputs and outputs.
- ▶ Compare the counterfactual to the baseline scenario:
 - ▶ A direct impact via average productivity;
 - ▶ An indirect impact via intra-firm resource reallocation.

How Costly is Quality?

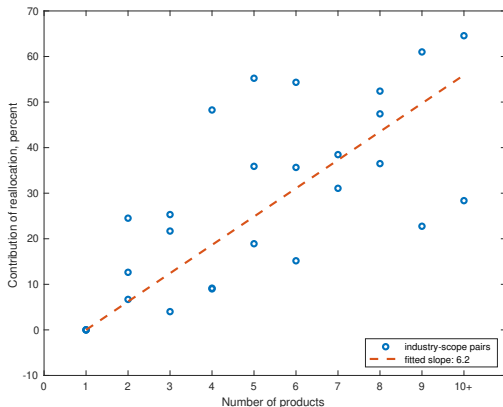
Table: Impact of 1% reduction in cost of quality on ATFP

| Industry | All | Footwear | All firms | | MPF only All |
|-------------------------------------|------------------|------------------|------------------|------------------|------------------|
| | | | Printing | Pharmaceutical | |
| Total improvement, percent | 2.635 (0.282) | 0.851 (0.287) | 2.739 (0.382) | 2.791 (0.339) | 2.754 (0.308) |
| Intra-firm reallocation, percent | 0.698 (0.123) | 0.062 (0.030) | 0.447 (0.116) | 0.795 (0.151) | 0.815 (0.142) |
| percentage relative to total | 26.5 (5.1) | 7.3 (2.2) | 16.3 (3.9) | 28.5 (6.1) | 29.6 (5.8) |

Observations:

- ▶ Complementary to the literature emphasizing cross-firm reallocation, the contribution of **intra-firm resource reallocation** to firm overall ATFP is also sizable.
- ▶ Relative contribution is larger in industries with more products.

Intra-firm Reallocation and Product Scope



Observation:

- ▶ Larger scope allows for more room for **intra-firm resource reallocation** – a new mechanism for enhancing the performance of multi-product firms.

Conclusion

A new method to estimate productivity and quality at the firm-product level.

- ▶ **Methodology**: uncover productivity and quality (instead of input shares);
- ▶ **low data requirement**: accommodate (unobservable) intra-firm input sharing and heterogenous material prices;
- ▶ **scalability**: applicable to industries with many products.

Application to Mexico data uncovers productivity/quality and their relationship with intra-firm heterogeneity:

- ▶ core products have higher productivity and quality, however, there is a trade-off between the two (i.e., **cost of quality**).
- ▶ after taking both benefit and cost of quality into account, **ATFP** is positively associated with quality.
- ▶ reducing the cost of quality significantly improves ATFP, a sizable portion of which is contributed by **resource reallocation within firms** – a benefit of being a multi-product firm.

Product list: Footwear

Table: Product list, manufacturing of footwear, mainly of leather (class 324001)

| Industry | Product description | Code |
|----------|------------------------|------|
| 324001 | Cow leather, for men | 1 |
| 324001 | Cow leather, for women | 2 |
| 324001 | Cow leather, for kids | 3 |
| 324001 | Others | 99 |

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Product list: Printing

Table: Product list, printing and binding (class 342003)

| Industry | Product description | Code |
|----------|--|------|
| 342003 | Printing of Calendars and almanacs | 5 |
| 342003 | Folding boxes | 6 |
| 342003 | Labels and prints | 13 |
| 342003 | Brochures and catalogs | 14 |
| 342003 | Continuous forms | 15 |
| 342003 | Accounting, administrative and tax forms | 16 |
| 342003 | Telephone directories | 17 |
| 342003 | Books | 18 |
| 342003 | Journals | 19 |
| 342003 | Checks | 21 |
| 342003 | Commemorative and business cards | 23 |
| 342003 | Commercial flyers | 24 |
| 342003 | Posters | 25 |
| 342003 | Others | 99 |

Product list: pharmaceutical

| Industry | Product description | Code |
|----------|---|------|
| 352100 | Medicinal products, for human use with specific action, anti-infectious: Bactericides | 11 |
| 352100 | Antiparasitics | 13 |
| 352100 | Dermatological | 15 |
| 352100 | Other products with specific action not included in other categories | 19 |
| 352100 | Medicinal products for human use for specialties with action on: Circulatory system | 21 |
| 352100 | Digestive system and metabolism | 22 |
| 352100 | Human musculoskeletal system | 23 |
| 352100 | Respiratory system | 24 |
| 352100 | Sensory organs | 25 |
| 352100 | Genitourinary organs, except hormones | 26 |
| 352100 | Blood and hematopoietic organs | 27 |
| 352100 | Central nervous system | 28 |
| 352100 | Hormones | 32 |
| 352100 | Vitamins and Vitamin Compounds | 43 |
| 352100 | Non-therapeutic products | 59 |
| 352100 | Others | 99 |

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What Determines Within-Firm Heterogeneity?

Table: Product rank (sales growth), and growth in productivity and quality

| Dep. var.: | (1) | (2) | (3) | (4) |
|--------------------------|----------------------|----------------------|----------------------|----------------------|
| Log product rank, growth | All | Footwear | Printing | Pharmaceutical |
| Δ Productivity | -1.564*** (0.278) | -2.667*** (0.770) | -1.470*** (0.321) | -1.720*** (0.558) |
| Δ Quality | -0.409*** (0.106) | -0.326*** (0.089) | -0.351*** (0.111) | -0.534*** (0.162) |
| Firm-Year FE | yes | yes | yes | yes |
| Product-Year FE | yes | yes | yes | yes |
| Observations | 8311 | 307 | 2448 | 5556 |
| R-squared | 0.541 | 0.683 | 0.686 | 0.485 |

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Cost of Quality and Product Differentiation

Cost of quality is higher for more differentiated products (measured as higher markup or lower elasticity of demand), consistent with prediction by Eckel, Iacovone, Javorcik, and Neary (2015).

| Dep. var.: Productivity | (1) IV | (2) IV | (3) IV |
|------------------------------|----------------------|---------------------|----------------------|
| Quality | -0.300*** (0.038) | 0.315 (0.215) | -0.071 (0.045) |
| Quality $\times \eta$ | 0.021*** (0.007) | | |
| Quality \times Markup | | -0.400** (0.171) | |
| Quality \times Markup, log | | | -0.516*** (0.195) |
| Firm-Product FE | yes | yes | yes |
| Firm-Year FE | yes | yes | yes |
| Product-Year FE | yes | yes | yes |
| Observations | 8160 | 8160 | 8160 |
| R-squared | 0.859 | 0.853 | 0.854 |
| Kleibergen-Paap F | 60.461 | 56.377 | 56.565 |
| Hansen J | 15.885 | 17.948 | 17.681 |

Cost of Quality and Product Age

The cost of quality decreases as the firm produces its products for a longer time.

| Dep. var.: Productivity | (1) IV | (2) IV |
|-------------------------|----------------------|----------------------|
| Quality | -0.207*** (0.045) | -0.207*** (0.045) |
| Quality × Age, log | 0.011* (0.006) | 0.011* (0.006) |
| Age, log | -0.030 (0.101) | -0.029 (0.099) |
| Firm-Product FE | yes | yes |
| Firm-Year FE | yes | yes |
| Product-Year FE | yes | yes |
| Observations | 8160 | 8160 |
| R-squared | 0.831 | 0.831 |
| Kleibergen-Paap F | 53.798 | 43.617 |
| Hansen J | 5.168 | 4.648 |

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